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Probabilities, Odds, and Forecasts of Rare Events

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ABSTRACT

Several issues related to the mode of expression of forecasts of rare and severe events (RSEs) are addressed in this paper. These issues include the correspondence between forecasters' judgments and their forecasts, the problem of overforecasting, and the use of forecasts as a basis for rational decision making. Neither forecasters nor users are well served by current practices, according to which operational forecasts of RSEs are generally expressed in a categorical format.

It is argued here that sound scientific and economic reasons exist for expressing forecasts of RSEs in terms of probabilities. Although quantification of uncertainty in forecasts of RSEs—and the communication of such information to users—presents some special problems, evidence accumulated from a multitude of operational and experimental probabilistic weather forecasting programs suggests that these problems involve no insurmountable difficulties. Moreover, when a probabilistic format is employed, forecasts of RSEs can correspond to forecasters' true judgments, the forecasting and decision-making tasks can be disentangled, the rationale for overforecasting RSEs is eliminated, and the needs of *all* users can be met in an optimal manner.

Since the probabilities of RSEs seldom achieve high values, it might be desirable to provide users with information concerning the likelihood of such events relative to their climatological likelihood. Alternatively, the relative odds—that is, the ratio of an event's forecast odds to its climatological odds—could be reported. This supplemental information should help to focus users' attention on those occasions on which the probability of RSEs is *relatively* high.

1. Introduction

Rare events that occur on relatively small space and time scales present weather forecasters with a very difficult scientific challenge, in terms of formulating space/time specific and skillful forecasts. Many such events involve severe weather conditions (e.g., tornadoes, severe thunderstorms, high winds, heavy precipitation) that significantly—and generally adversely—affect human activities and frequently lead to loss of life, injury, and/or property damage. Clearly, forecasts of these conditions are of considerable importance from a societal viewpoint.

Operational forecasts of rare and severe events (usually referred to here as RSEs) are traditionally expressed in a categorical format. For example, watches issued by the National Severe Storms Forecast Center (NSSFC) identify areas in which tornadoes or severe thunderstorms are expected to occur in a specified period. Of particular interest here, the uncertainty inherent in such forecasts—uncertainty that characterizes the likelihood of occurrence of the events and that presumably varies from occasion to occasion—generally remains unspecified.[1]

The thesis advanced in this paper is that the use of a categorical mode of expression complicates the process of forecasting RSEs and depreciates the product of the forecasting process (i.e., the forecasts). Specifically, this practice leads to hedging (i.e., differences between forecasters' judgments and their forecasts), overforecasting, and suboptimal decision making by users of the forecasts. These problems cannot be resolved satisfactorily as long as the forecasts are expressed in a categorical format. However, a scientifically sound and operationally realizable solution exists—namely, the formulation and expression of forecasts of RSEs in a quantitative probabilistic format.

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[1] According to official guidelines, a NSSFC watch indicates that events of significant spatial and/or temporal dimensions are *possible* in the specified area. However, no quantitative definition of the term "possible" is provided. Moreover, according to current operational verification procedures, a watch is considered to be *completely* correct when such events occur in the area (i.e., it is verified as a categorical forecast).

This short paper addresses issues related to the categorical mode of expression currently used in forecasts of RSEs, as well as issues related to the possible use of a probabilistic mode of expression in this context in the future. Rare events and rare-event forecasts are briefly described in section 2. Hedging is considered in section 3, the problem of overforecasting is examined in section 4, and decision making by users of forecasts of RSEs is discussed in section 5. Problems related to the use of a probabilistic mode of expression in forecasts of RSEs are discussed in section 6, and some suggestions regarding the use of probabilities and odds in such forecasts are described and illustrated in section 7. Section 8 consists of a brief summary as well as some concluding remarks.

2. Rare events and rare-event forecasts

For the purposes of this paper, a rare event is defined as an event that occurs on less than 5% of the forecasting occasions. Thus, if π denotes the climatological probability of the RSEs of interest here, then by definition $\pi < 0.05$[2]. In addition, it is sometimes useful to consider the odds in favor of or against such events. The odds in favor of an event are simply the ratio of the probability of the event to the probability of the complementary event. Therefore, the climatological odds in favor of RSEs are $\pi/(1-\pi) < 1/19$. The odds against a rare event are the reciprocal of the odds in favor of the event; the climatological odds against RSEs are thus $(1-\pi)/\pi > 19/1$.

The ability to predict RSEs is limited for a variety of reasons that are well-known to operational weather forecasters (e.g., the small spatial and temporal dimensions of the events). As a result, forecasts of such events exhibit a relatively modest degree of skill (Doswell and Flueck 1989; Doswell et al. 1990; Weiss et al. 1980). Here, skill refers to the accuracy of the forecasts relative to the accuracy of forecasts based solely on the relevant climatological probabilities. In addition, forecasts of RSEs generally exhibit moderate to substantial amounts of overforecasting. That is, the frequency with which RSEs are forecast usually exceeds—sometimes by a factor of two or more—the frequency with which these events are observed (Doswell and Flueck 1989; Doswell et al. 1990; Weiss et al. 1980).

3. Hedging: Judgments and forecasts

In the context of subjective weather forecasting, forecasters formulate judgments regarding the likelihood of occurrence of the events of interest and then translate these judgments into forecasts. Since the judgments represent the forecasters' true beliefs, they are inherently probabilistic (Murphy 1985). The complex process of evaluating and assimilating information from a variety of sources, and incorporating this information into conceptual (and other) models, seldom enables forecasters to reach unqualified categorical judgments regarding event occurrence or nonoccurrence.

As noted in section 1, however, forecasts of RSEs are traditionally expressed in a categorical format. It is evident, then, that these forecasts generally do *not* correspond to the forecasters' judgments. Situations in which forecasts differ from judgments are said to involve *hedging* (Murphy and Winkler 1971).

At this point, it is useful to distinguish between *internal forecasts* formulated within a forecast office and *external forecasts* communicated to users. Since internal forecasts by definition are isolated from considerations related to their use, little if any justification exists for not making these forecasts correspond to the respective judgments. This objective, and the concurrent elimination of hedging, can be realized only by expressing the forecasts in quantitative probabilistic terms. The unavoidable conclusion that must be drawn from this argument was succinctly summarized many years ago by Sanders (1963) when he stated that probability should become "the internal language of forecasters."

In forecasting situations involving RSEs, the case for expressing (internal) forecasts in terms of probabilities would appear to be especially strong. Since these events are relatively infrequent and extremely difficult to forecast, the likelihood of occurrence of rare events is presumably quite low on a vast majority of forecasting occasions. Moreover, on the relatively small number of occasions on which such events are considered to be somewhat more likely to occur, forecasters are seldom in a position to make unqualified categorical statements regarding their occurrence. In such situations, the binary vocabulary of categorical forecasts ("yes" or "no") is particularly inappropriate. On the other hand, the relatively rich (and essentially continuous) vocabulary of probabilistic forecasts provides a means of describing scientifically justifiable, and operationally meaningful, differences in a forecaster's judgments from occasion to occasion.

Note that expressing (internal) forecasts in terms of probabilities actually would simplify the forecasting process. It would no longer be necessary for forecasters to devote any time or effort to the process of translating probabilistic judgments into categorical forecasts—a non-trivial exercise involving (*inter alia*) user-related considerations. Instead, the forecaster's judgments would simply be "relabelled" as forecasts.

4. Overforecasting

Currently, forecasters translate their (probabilistic) judgments into external forecasts of RSEs expressed in

[2] It should be noted that not all severe weather events (as defined operationally) necessarily belong to the class of RSEs considered here.

a categorical format. In performing this task, the forecasters must choose between two alternatives—for example, "watch" or "no watch"—and this choice is evidently influenced by both their judgments concerning the likelihood of occurrence of the events *and* their perceptions regarding the relative impacts on users of the two possible types of forecast errors. These errors are a) a forecast of "watch" followed by event nonoccurrence (type 1 error) and b) a forecast of "no watch" followed by event occurrence (type 2 error). Since it is widely perceived that type 2 errors are more serious than type 1 errors, forecasts of RSEs generally are characterized by overforecasting. That is, over a set of forecasting occasions, more RSEs usually are forecast to occur than are subsequently observed to occur. The amount of overforecasting associated with forecasts of some RSEs is quite substantial (e.g., see Doswell and Flueck 1989; Doswell et al. 1990; Weiss et al. 1980), and efforts to reduce this overforecasting—as well as attempts to prescribe an appropriate or acceptable amount of overforecasting—have received considerable attention.

As long as forecasters must transform their probabilistic judgments into categorical forecasts, and as long as they perceive that type 2 errors are more serious than type 1 errors, their forecasts will necessarily exhibit some degree of overforecasting. The amount of overforecasting will be directly related to the forecasters' perceptions (or assumptions) concerning the relative importance of these two types of errors, and the latter implicitly relates to the cost-loss functions of the users of the forecasts.[3] The basic issue here can be stated in the form of a question: How much overforecasting is appropriate? Since the appropriate amount of overforecasting depends on users' cost-loss functions, and the information available to forecasters concerning these cost-loss functions generally is extremely limited, it would seem that forecasters are not in a very good position to make an informed decision as to how to translate their judgments into categorical forecasts.

It is also important to recognize that different users usually possess different cost-loss functions. Individuals with different cost-loss functions frequently require different categorical forecasts (for optimal decision making). In general, a single categorical forecast cannot adequately reflect the spectrum of cost-loss functions associated with users of forecasts of RSEs.

Since it is impossible to identify an appropriate amount of overforecasting (except in the case of a single user with a well-defined cost-loss function), the only identifiable, realizable, and reasonable goal is *unbiased forecasting* (i.e., no overforecasting or underforecasting). This goal can be realized by forecasting a rare event with the same frequency as that with which it actually occurs—a goal that can be achieved by formulating reliable probabilistic forecasts of the event. Such forecasts should correspond to the forecaster's best judgments and should not be influenced by considerations related to the users' cost-loss functions. A probabilistic format for forecasts of RSEs separates the forecasting and decision-making tasks—the former is performed by the forecaster to the best of his/her ability, and the latter is performed by the user who presumably is better equipped than the forecaster to accomplish this task.

5. Decision making by users of forecasts

In order to make optimal (i.e., the best possible) decisions, individuals whose choices among alternative courses of action are influenced by a weather forecast need to know the likelihood of occurrence of the relevant events given the forecast (Winkler and Murphy 1985). Thus, decision makers provided with categorical forecasts of RSEs face a dilemma. In essence, two options are available to such individuals. They can take the forecast at face value, in which case it necessarily leads to one and only one decision; namely, the alternative that is best when the forecast event subsequently occurs. In this case, uncertainty is ignored and, in effect, the forecaster "becomes" the decision maker, a role for which he/she generally is not well-equipped.

Alternatively, since decision makers often recognize that categorical forecasts are not always correct, they may try to take this fact into account. However, a categorical forecast provides no information regarding the probabilities of occurrence of the various events. Of course, the user can try to calibrate the forecast by taking into account the forecaster's past performance—assuming that the relevant data are available or that the user is willing to trust his intuition in this regard. It should be noted, however, that conditional probabilities derived from such a calibration process may be largely irrelevant to the particular situation at hand.

Reliable probabilistic forecasts contain all of the information needed by users to make optimal decisions (if these forecasts are not reliable, then they too need to be calibrated). Moreover, it is relatively easy to show that the value of such forecasts equals or exceeds the value of categorical forecasts for all users (Krzysztofowicz 1983; Murphy 1977; Thompson 1962). Here, forecast value is measured in terms of the costs and/or losses associated with users' decision-making problems.

At present, little if any information exists regarding the efficiency, or effectiveness, of users' decision-making processes (whatever the format of the forecasts).

[3] As decision makers, *all* users of weather forecasts possess a cost-loss function. This function reflects the users' preferences for the possible outcomes, each of which corresponds to a particular combination of actions and events (Winkler and Murphy 1985). In some cases these preferences largely reflect economic (i.e., monetary) considerations, but in other cases (e.g., situations involving RSEs) they involve both economic and non-economic considerations.

Nevertheless, the fact that users' need situation-specific and reliable probabilistic forecasts to make optimal decisions suggests that they may not be well-served by categorical forecasts of RSEs. The information contained in *probabilistic* forecasts of RSEs should improve the efficiency of the decision-making process, yield better decisions, and enhance forecast value. Studies of users' weather-information-sensitive decision-making problems will be required to realize the full extent of these benefits.

6. Probabilistic forecasts of RSEs

Since this paper argues that forecasts of RSEs should be expressed in probabilistic terms, it is appropriate here to discuss briefly some of the issues that frequently arise in connection with the formulation and communication of such forecasts. These issues include a) the quantification of uncertainty in forecasts, b) the mode of expression of uncertainty, c) the interpretation (or misinterpretation) of probabilistic forecasts, and d) the preferences of users regarding the format of forecasts. For a general and more comprehensive discussion of these and other issues, see Murphy (1985) and the references cited therein.

Although most forecasts of RSEs are expressed in a categorical format, some rare-event forecasts have been, or are routinely, formulated in a probabilistic format (e.g., Doswell and Flueck 1989; Ivarsson et al. 1986; Murphy and Daan 1984; Murphy et al. 1985; Murphy and Winkler 1982). Two problems frequently arise when forecasters formulate probabilistic forecasts of RSEs. First, subjective probabilistic forecasts of such events may not be very reliable, in the sense that the forecast probabilities may not correspond closely to the observed relative frequencies. Lack of reliability is often particularly evident in the early stages of new probability forecasting programs. Several reasons for this lack of reliability can be identified, including the forecasters' inexperience in quantifying the uncertainty in their forecasts and the absence of timely and meaningful feedback regarding individual forecasting performance. Experience and feedback can help to overcome this problem (e.g., see Murphy and Daan 1984), as can the availability of objective probabilistic forecasts as guidance. In the case of RSEs, learning from feedback and experience may prove to be a more difficult and time-consuming process, because these events occur relatively infrequently and because of delays in the receipt of reliable verification data.

Second, since the relative frequency of occurrence of RSEs is very low, the range of probability values actually used in the forecasts may be quite limited. In particular, relatively high probability values (e.g., greater than 0.5) may be used very infrequently. It is important to recognize that the distribution of the frequency of use of probability values is simply a reflection of the state-of-the-art of forecasting RSEs (i.e., the predictability of such events). When the predictability is low, as it is for many of these events at present, this distribution will be a fairly tight unimodal distribution centered at the climatological probability (assuming overall reliability). Large excursions from the climatological probability necessarily will be quite infrequent. As predictability improves (or for events for which the predictability is already markedly greater), this distribution will become bimodal (i.e., u-shaped) with peaks at or near probabilities of zero and one.

Many weather forecasts, including some forecasts of RSEs, are expressed in a qualitative probabilistic format (e.g., NSSFC outlooks). That is, the forecasts contain words that are intended to describe the uncertainty associated with event occurrence in qualitative terms. Although such forecasts contain situation-specific information concerning uncertainty, and different words may lead to different decisions (the user becomes the decision maker once again), this format places a considerable burden on users of the forecasts. They must be able to interpret the words correctly (i.e., according to the forecaster's definitions) and translate these qualitative expressions of uncertainty into information that provides a basis for rational decision making. Unfortunately, considerable evidence exists that the words used to describe uncertainty in forecasts (e.g., "chance," "likely") are subject to such wide ranges of interpretation (by users) as to render this mode of expression largely incapable of providing useful information (Murphy and Brown 1983).

Unlike verbal expressions of uncertainty, probabilities provide a precise and unambiguous description of the uncertainty in forecasts. Nevertheless, it has been argued that users frequently misinterpret probabilistic forecasts. In the case of precipitation probability forecasts, for example, studies have shown that users frequently interpret such forecasts as area or areal-coverage forecasts rather than as point forecasts. In reality, however, misinterpretation of probabilistic forecasts is due almost entirely to event misinterpretation—a problem common to *all* types of forecasts—rather than to probability misinterpretation (Murphy et al. 1980).

Finally, it should be noted that the public now prefers precipitation probability forecasts to verbal expressions of the uncertainty in these forecasts by a ratio of two or three to one (M.S.I. Services, Inc. 1981). Moreover, precipitation probabilities are now disseminated to the general public in many countries. It also should be noted that the official forecasts of hurricane movement produced by the National Hurricane Center are now accompanied by probabilities that indicate the likelihood that specified locations along the coastline of the United States will experience hurricane force winds (Sheets 1985).

7. Probabilities and odds in forecasts of RSEs

As noted in section 6, the probabilities assigned to RSEs are seldom very high. For example, for a rare event with a climatological probability of 0.05, (reliable) forecast probabilities may seldom exceed 0.50. Some users may tend to ignore such relatively modest probability values. Thus, in communicating such forecasts to users, it might be beneficial to translate the probabilities into odds for or against the events, or to provide users with information in the form of odds in addition to the probabilities. To illustrate this concept, some specific examples are briefly considered.

As before, let π denote the climatological probability of the RSE of interest, and let p denote the forecast probability on the particular occasion of concern. Consider a situation in which $\pi = 0.05$ and $p = 0.25$. The occurrence of the event on this occasion is less likely than its nonoccurrence, but it is considerably more likely "today" than it is climatologically. Specifically, it is five times more likely ($= p/\pi = 0.25/0.05 = 5/1$) today than it is "on the average." It is the relative magnitudes of p and π that are important here, not whether or not p exceeds some arbitrary threshold value such as 0.5. Thus, the forecaster may want to include the ratio p/π, as well as p, in the forecast provided to users.

It is even more instructive to consider the odds in this case. The climatological odds (CO) in favor of the event are $\pi/(1 - \pi) = 1/19$ and the forecast odds (FO) in favor of the event are $p/(1 - p) = 1/3$. Thus, even though the odds on this occasion favor the common event, FO is considerably larger than CO. This change in odds (from CO to FO) can be quantified in terms of the odds ratio (OR), where OR = FO/CO. In this case, OR = 19/3. The forecast odds in favor of the event are more than six times greater than the climatological odds. It is important to recognize that OR reflects both the increase in the probability of the rare event (from π to p) *and* the decrease in the probability of the complementary event (from $1 - \pi$ to $1 - p$). The latter is not considered when only the ratio p/π is evaluated (as in the previous paragraph).

The difference between OR and p/π is even greater for less likely events. For example, suppose that $\pi = 0.01$ and $p = 0.25$. Then $p/\pi = 25/1$, but OR is 33/1. In this case, the RSE is 25 times more likely today than it is on the average (i.e., from a climatological point of view), but the forecast odds in favor of the RSE are 33 times greater than the climatological odds. Inclusion of this information in the forecast may "overcome" the relatively modest probability values and enable users to identify those occasions on which the risk of RSEs is sufficiently high to warrant taking appropriate precautionary (or protective) measures.

8. Summary and conclusion

Traditional practices with respect to the mode of expression of forecasts of RSEs can be called into question for several reasons. First, despite the fact that forecasters' judgments generally exhibit considerable uncertainty regarding the occurrence (or nonoccurrence) of RSEs, their categorical forecasts of these events do not reflect this uncertainty. Second, the use of a categorical format for such forecasts—when combined with forecasters' usual perceptions concerning the relative impacts of the two possible types of forecast errors—leads to substantial overforecasting of RSEs. In the process, forecasters must make inferences regarding the cost-loss functions of users (a difficult if not impossible task under the circumstances) and the forecasts necessarily are tailored to a particular subset of users. Moreover, overforecasting adversely affects the overall credibility of the forecasts (via the "cry wolf" syndrome) and depreciates their utility to users. Finally, these practices and their consequences create a situation in which most users do *not* receive the information that they need to make the best possible decisions.

The arguments set forth in previous sections of this paper make a strong case for formulating, expressing, and communicating forecasts of RSEs in a probabilistic format. Such a format would make it possible for official forecasts of RSEs to correspond to forecasters' true judgments, thereby eliminating hedging. In addition, expressing forecasts of RSEs in terms of probabilities provides a basis for establishing the realizable goal of unbiased forecasting, thereby disentangling the forecasting and decision-making tasks. As a result, it should be possible to greatly reduce (if not eliminate) the overforecasting that currently plagues forecasts of RSEs and to relieve forecasters of the difficult task of making inferences regarding users' cost-loss structures, a task for which they are not well-equipped. Moreover, forecasts of RSEs expressed in a probabilistic format contain the information that *all* users need to make rational and optimal decisions.

Forecasters have gained considerable experience in the quantification of uncertainty in weather forecasts over the last 25 years. Reliable and skillful probabilistic forecasts of precipitation occurrence, and other variables, are now formulated on an operational basis in many countries. As noted in section 6, probabilistic forecasts of RSEs present a particular challenge, in that formulating reliable forecasts and learning from experience may be more difficult and time-consuming in this context. Nevertheless, experimental and limited operational programs in probabilistic rare-event forecasting indicate that these problems are not insurmountable. Of course, forecasters should make such forecasts on an experimental basis prior to their operational implementation. In addition, an ongoing program designed to educate potential users regarding the interpretation and use of probabilistic forecasts of RSEs would be an essential component of any such effort.

In summary, forecasts of rare events should be expressed in a probabilistic format. Probability, as the language of uncertainty, provides a quantitative framework within which the likelihood of such events can be assessed and within which forecasters can evaluate the impact (on these likelihoods) of additional data or information in a rational and unambiguous manner. Moreover, forecasters should consider supplementing their probabilistic forecasts of RSEs with information consisting of the likelihood of the events relative to their climatological likelihood (i.e., p/π) or the ratio of the forecast odds to the climatological odds (i.e., OR). This additional information should be helpful in alerting users to the risks of RSEs on specific occasions and in identifying the optimal responses to forecasts of such events.

Finally, it has been argued here that a probabilistic mode of expression is most appropriate for forecasts of *rare and severe events*. It should be noted that this same argument also applies to forecasts of other weather events, regardless of their likelihood of occurrence or severity. The problems related to the correspondence between judgments and forecasts, overforecasting (or underforecasting), and optimal decision making by users can be resolved in a scientifically sound and operationally meaningful way by expressing the respective forecasts in quantitative, probabilistic terms.

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