

Strongly Coupled Data Assimilation with a Linear Inverse Model

[using emulators to rapidly prototype coupled-DA strategies for S2S forecasting]

Gregory Hakim¹, Lindsey Taylor¹, Chris Snyder²,
Matthew Newman³, and Steven Penny⁴

¹University of Washington, ²NCAR, ³NOAA ESRL – NOAA/ESRL, ⁴Sofar Ocean Technologies

Research sponsored by NOAA NWS/STI Award NA20NWS4680053

Strongly Coupled Data Assimilation (SCDA)

- Observation assimilation updates all modeled components of Earth system
- **SCDA**: Extremely computationally demanding (e.g. Liu et al. 2013; Penny & Hamill 2017)
 - Coupled forecast and “strongly coupled” DA: cross-medium updates
- “Weakly coupled” approximations (**WCDA**) (e.g. Saha et al. 2006; Zhang et al. 2007; Penny et al. 2019)
 - Separate DA in atmosphere & ocean with a coupled forecast step
 - No “cross covariance” influence from observations
 - Potentially “incompatible” states
 - Still very computationally demanding

Is SCDA Worth the Expense?

Need an efficient way to **estimate the benefits**

We propose a framework for this estimate using a **linear emulator**

- Low dimensionality promotes experimentation/prototyping
- Unlike “toy” models, the emulator skillfully forecasts observed fields
- Allows unapproximated Kalman filter, and WCDA-SCDA evaluation

Linear Inverse Models (LIMs)

e.g. Penland (1989); Newman et al. (2003); Breeden et al. (2020)

$$\frac{d\mathbf{x}}{dt} = \mathbf{L}(\mathbf{x}) \approx \mathbf{L}\mathbf{x} \quad \rightarrow \quad \mathbf{x}(\tau) = \mathbf{G}_\tau \mathbf{x}_0 + \boldsymbol{\epsilon} \quad \mathbf{G}_\tau = e^{\mathbf{L}\tau}$$

Solve for G empirically from sample data:

$$\mathbf{G}_\tau = \text{cov}(\mathbf{x}_\tau, \mathbf{x}_0) (\text{cov}(\mathbf{x}_0, \mathbf{x}_0))^{-1}$$

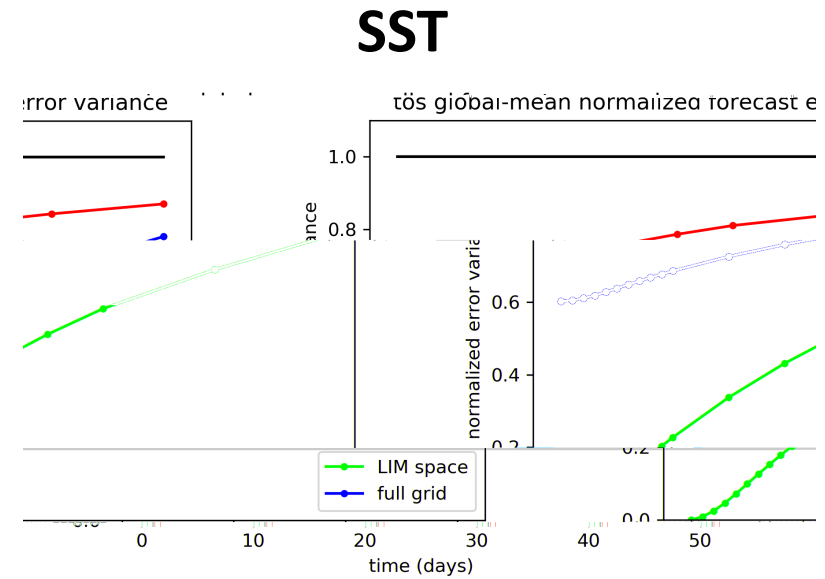
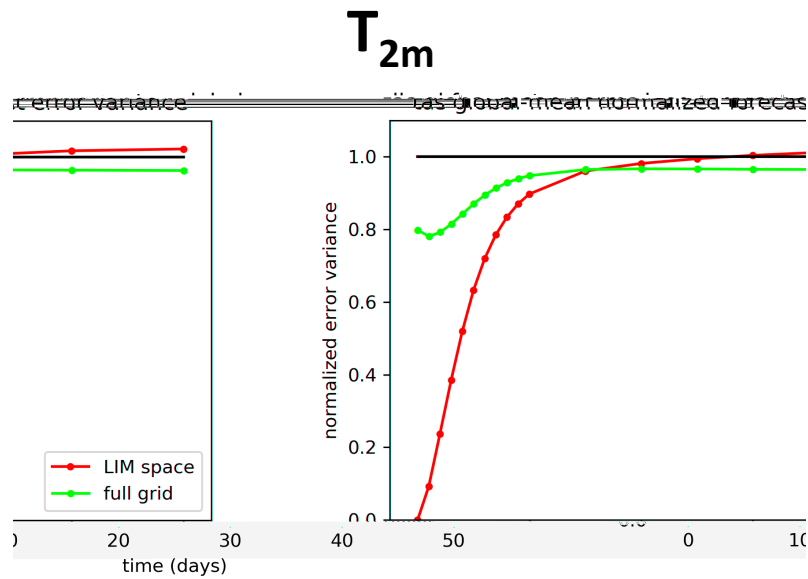
and the noise error-covariance matrix

$$\text{cov}(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}) = \mathbf{N}_\tau = \mathbf{C} - \mathbf{G}_\tau \mathbf{C} \mathbf{G}_\tau^T \quad \mathbf{C} = \text{cov}(\mathbf{x}_0, \mathbf{x}_0)$$

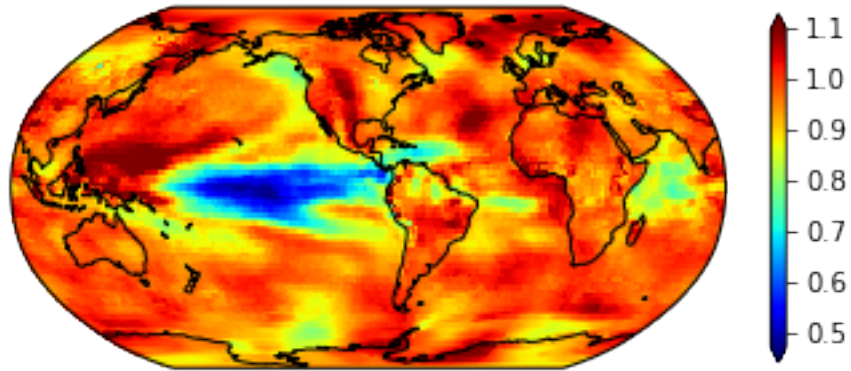
LIM Training

- Climate Forecast System Reanalysis (CFSR) (Saha et al. 2010)
- Global gridded fields of **2m air temperature** (T_{2m}), **SST**, 850 hPa **u** & **v**, and **OLR**
 - diurnal & 5-day running mean; seasonal cycle removed
- Truncate to leading 30 EOFs for each variable (150 degrees of freedom in total)
- LIM **training** period: 1979-2003 (9130 days)
- LIM **validation** period: 2004-2010 (2556 days)

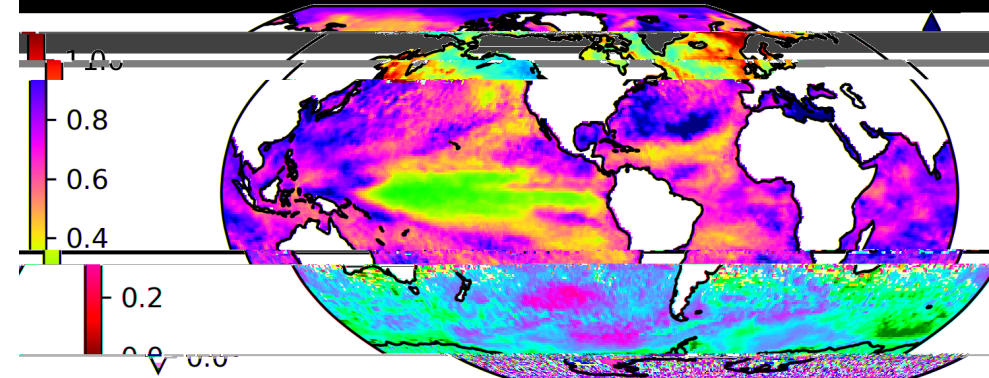
LIM forecast verification 2004-2010



tas normalized error variance lag=10 days



sst normalized error variance lag=10 days



Kalman Filter using the LIM

analysis

$$\mathbf{x}_a = \mathbf{x}_f + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}_f)$$

$$\mathbf{P}_a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_f$$

$$\mathbf{K} = \mathbf{P}_f^T \mathbf{H}^T [\mathbf{H} \mathbf{P}_f^T \mathbf{H}^T + \mathbf{R}]^{-1}$$

forecast

$$\mathbf{x}_f = \mathbf{G}_t \mathbf{x}_a$$

$$\mathbf{P}_f = \mathbf{G}_t \mathbf{P}_a \mathbf{G}_t^T + \mathbf{N}_t$$

full matrices!

Cycling time $t = 1$ day

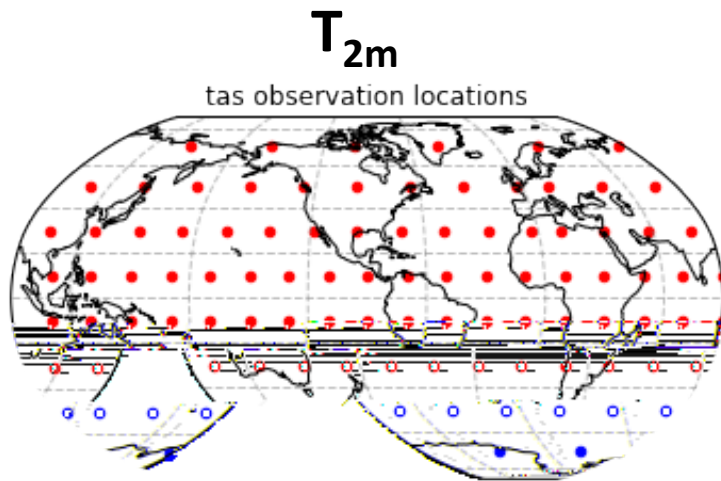
Pointwise observations from CFSR

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e} = \mathbf{H}^*\mathbf{U}\mathbf{x} + \mathbf{H}^*\boldsymbol{\eta}$$

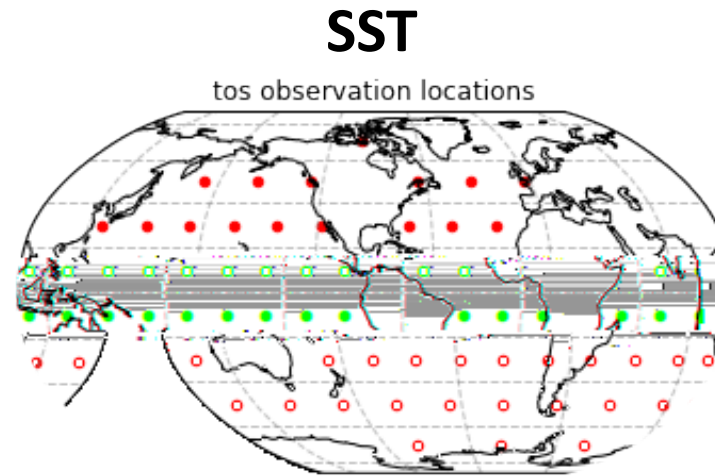
obs from LIM basis **truncation error**

\mathbf{H}^* : obs operator on CFSR grid

\mathbf{R} derived from truncation error



104 locations

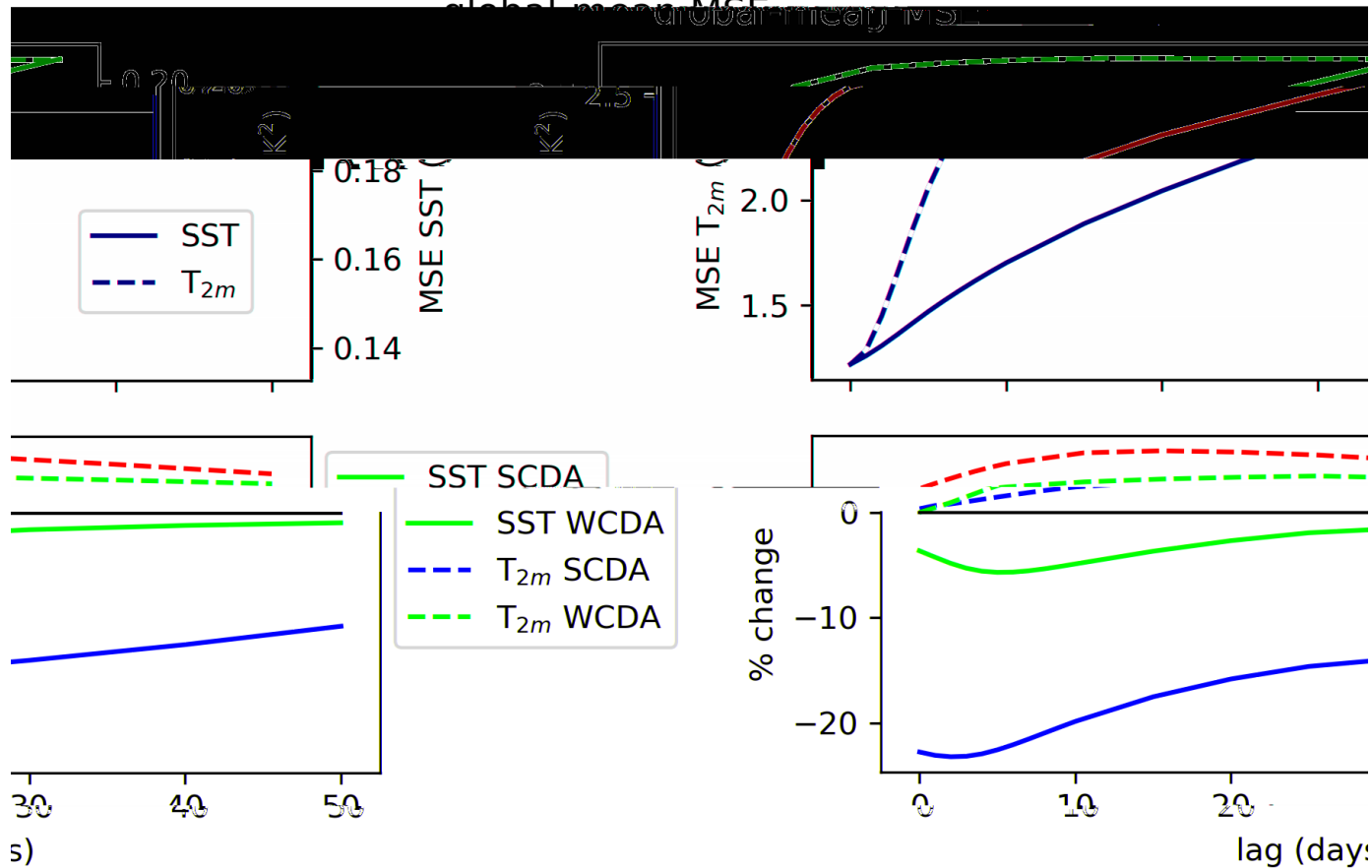


73 locations

DA Experiments

1. Uncoupled: **atmosphere**
 - Atmosphere components of the LIM & atmosphere observations
2. Uncoupled: **ocean**
 - Ocean components of the LIM & ocean observations
3. Weakly coupled (**WCDA**)
 - Full LIM; separate DA in atmosphere & ocean; no cross covariances
4. Strongly coupled (**SCDA**)
 - Full LIM; fully coupled DA

Global-mean Mean Squared Error



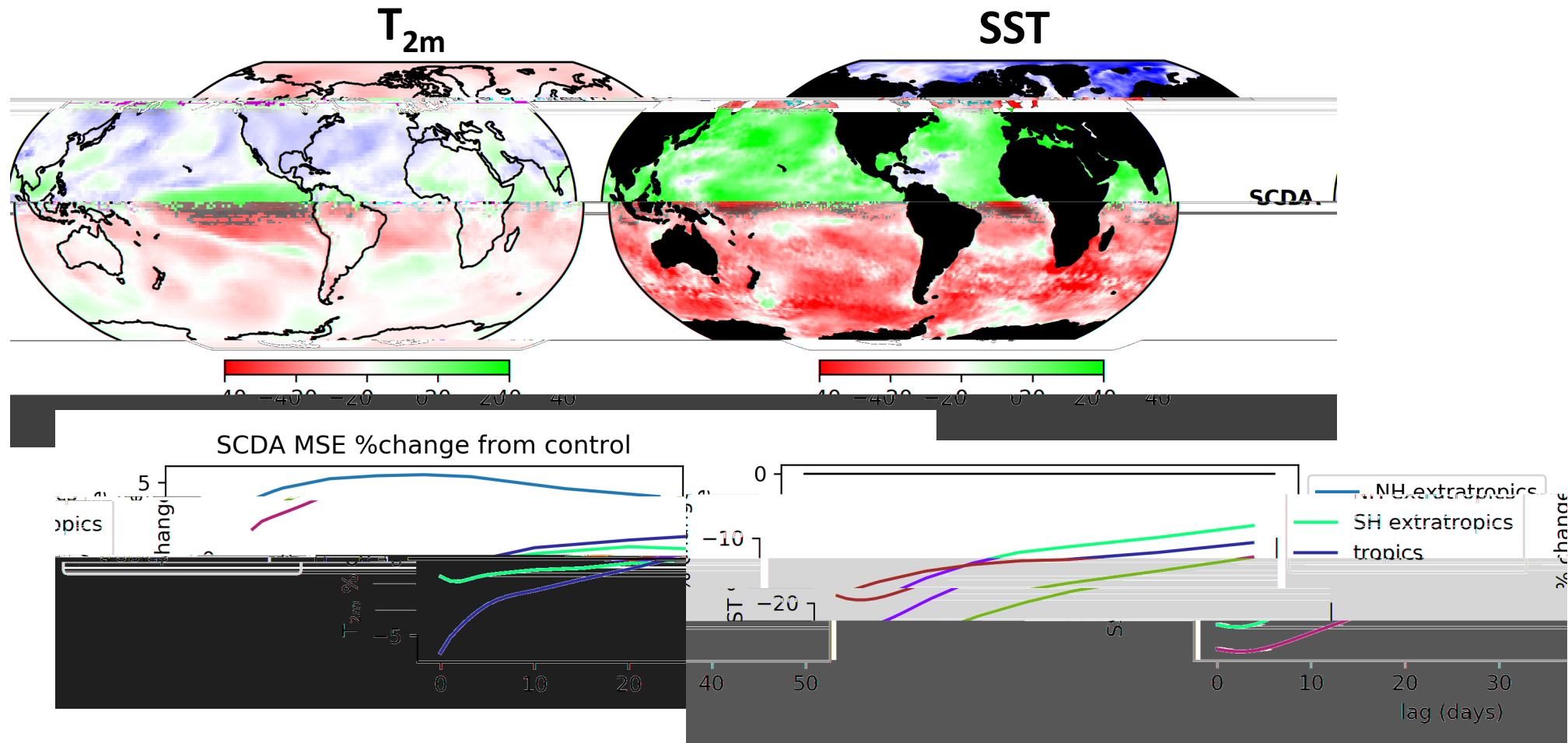
uncoupled (**control**) experiments

WCDA & SCDA

change from control experiments

- large positive impact on SST
- weak adverse impact on T_{2m}

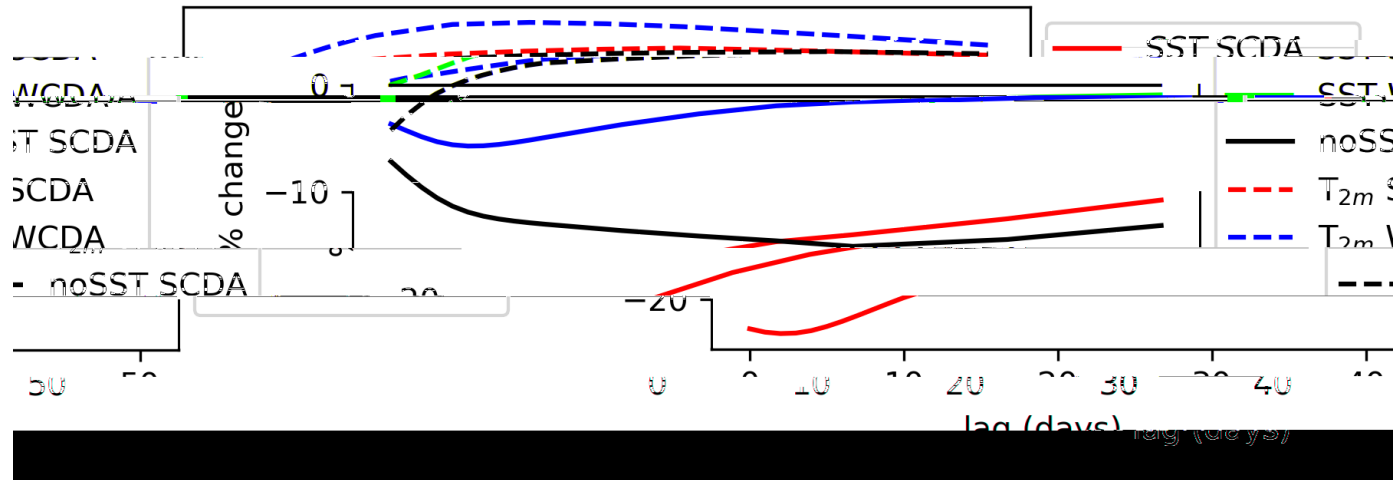
SCDA Change from Uncoupled 10-day Forecasts



- improved tropics
- degraded NH extratropics

- global improvement

SCDA without SST observations

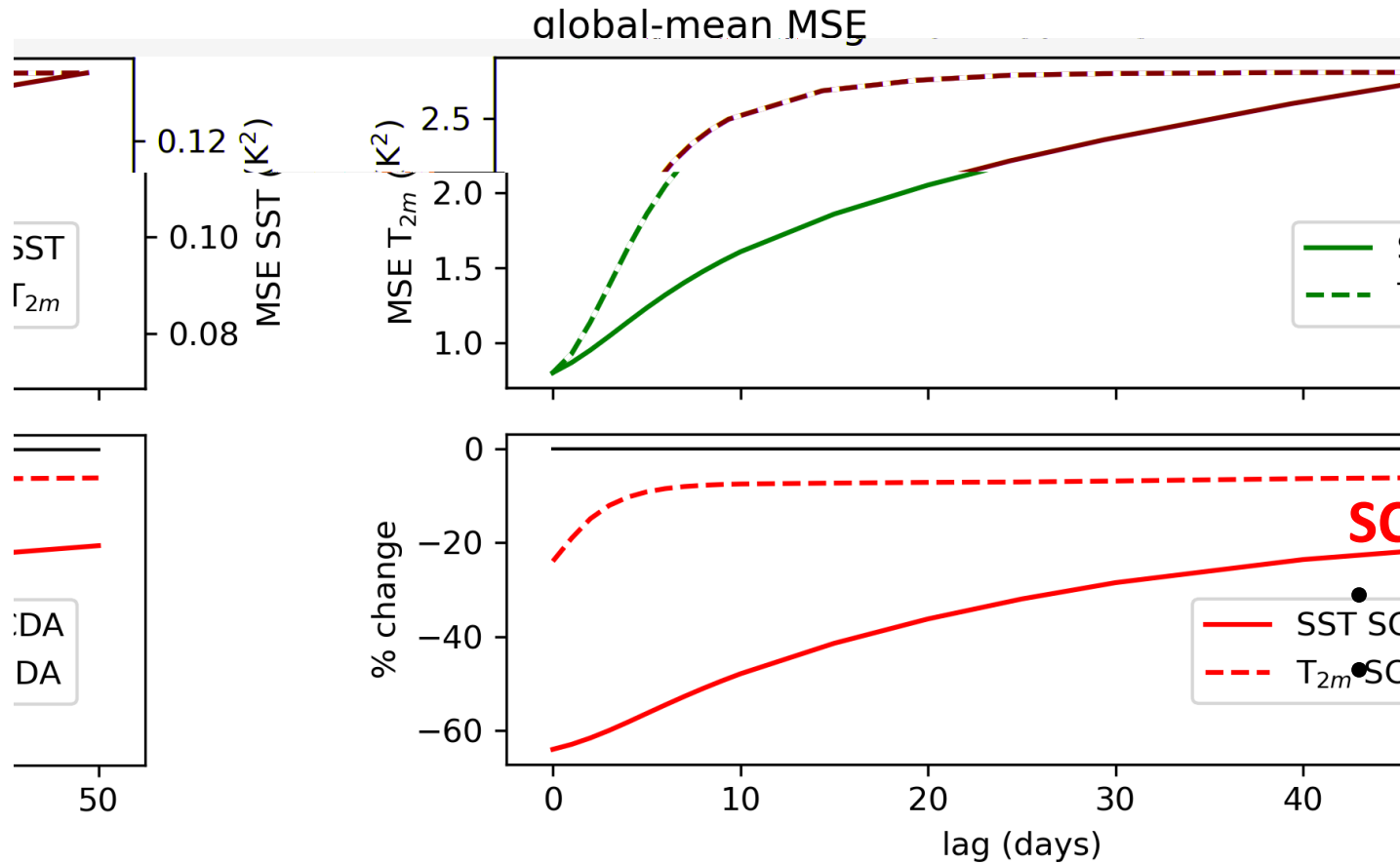


WCDA & SCDA

change from control experiments

- better T_{2m} analyses
- SST analyses still better than control
- long-lead forecasts similar

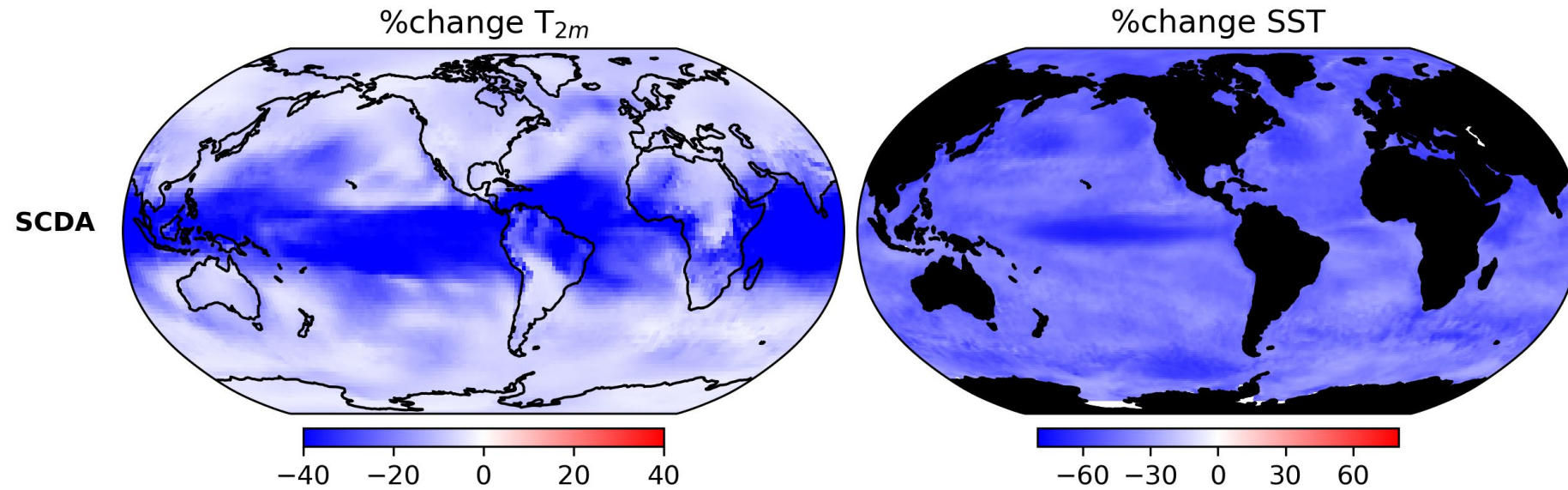
SCDA Results During the Training Period



uncoupled (**control**) experiments
 • smaller errors than validation

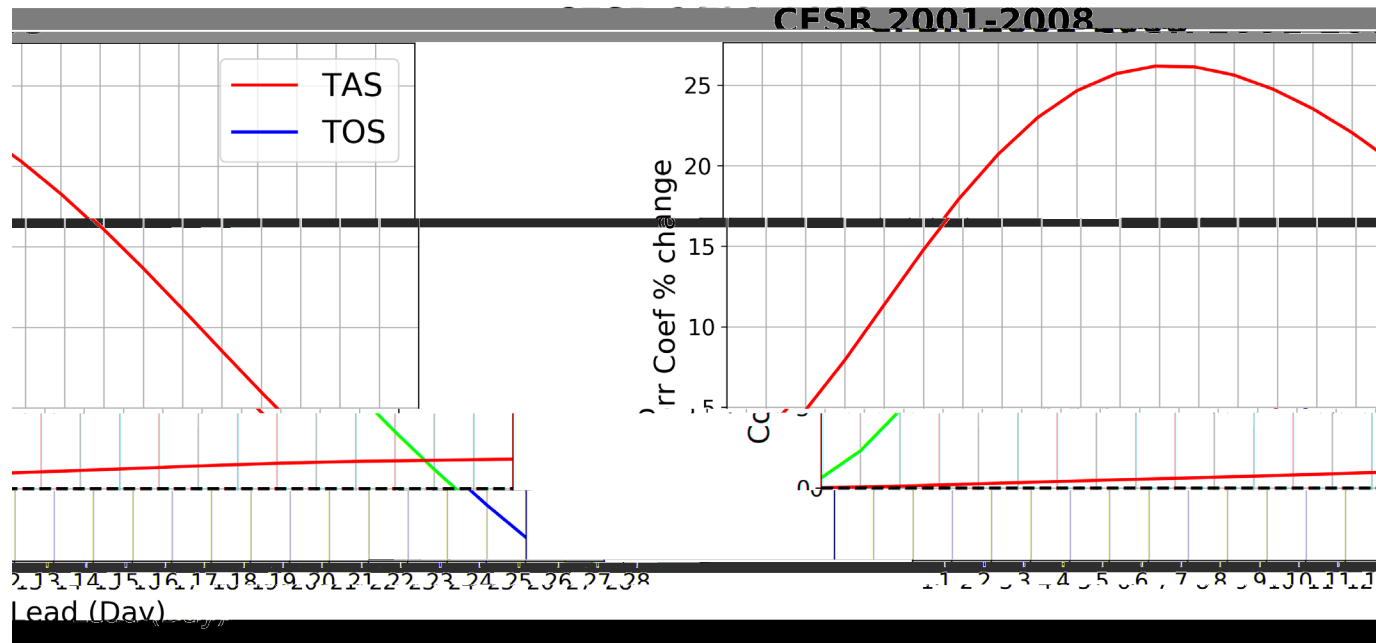
SCDA change from control experiment
 • large improvements
 • non-stationary climate

SCDA Training Period 10-day Forecast Change

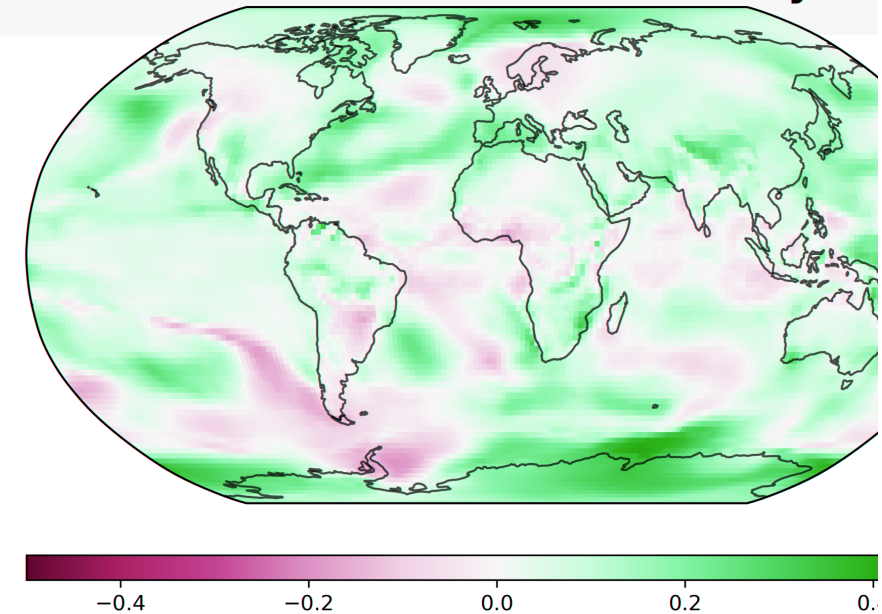


Spatial pattern of % change in MSE relative to the control case.

Experiments with larger training data: JRA55



TAS Corr Coef Difference: day 12



- JRA LIM trained 1958-2000
- CFSR LIM trained 1980-2000
- Validation: CFSR 2001-2008



Lindsey Taylor (UW)

Summary: LIM DA Proof of Concept

- **LIMs provide a flexible tool for coupled-DA hypothesis testing**
 - Kalman filtering, 4DVAR, ensemble strategies, etc.
- **SCDA performs better than WCDA in all experiments**
- **Relative to control, single-domain, experiments, SCDA:**
 - improves SST forecasts: ~10–25% better in global mean; 40+% locally
 - improves tropical T_{2m} forecasts; degrades extratropical NH T_{2m} forecasts
 - large T_{2m} improvement during training does not hold up in validation
- Currently exploring other LIM bases not truncated on EOFs; ensembles; seasonality

Hakim, G. J., C. Snyder, S. G. Penny, and M. Newman, 2022: Subseasonal forecast skill improvement from strongly coupled data assimilation with a linear inverse model. *Geophysical Research Letters*, **49**, e2022GL097996.

DOI: [10.1029/2022GL097996](https://doi.org/10.1029/2022GL097996)

Thank you!